Magnetorotational instabilities and pulsar kick velocities

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HIGHLIGHTS

- A model involving birth magneto-rotational instabilities (MRI) in neutron stars (NS) is suggested.
- A conversion of energy during a birth MRI is used to explain the NS kick velocities.
- Periods of ms and magnetic fields of 10^{16} G during a birth MRI yield the observed NS kick velocities.

ARTICLE INFO

ABSTRACT

At the end of their birth process, neutron stars can be subject to a magnetorotational instability in which a conversion of kinetic energy of differential rotation into radiation and kinetic energies is expected to occur at the Alfvén timescale of a few ms. This birth energy conversion predicts the observed large velocity of neutron stars if during the evolving of this instability the periods are of a few ms and the magnetic fields reach values of 10^{16} G.

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A major unsolved problem in astrophysics is to explain why neutron stars exhibit space velocities well above those of their progenitor stars (Anderson and Lyne, 1983; Chatterjee et al., 2005; Fryer, 2004; Hobbs et al., 2005; Lai, 2004; Lai et al., 2001; Postnov and Yungelson, 2006). Most neutron stars have been measured to possess high space velocities in the range of 100–1000 km/s, while their progenitor stars have velocities of the order of 10–20 km/s (Arzoumanian et al., 2002; Cordes and Chernoff, 1998; Fryer et al., 1998; Hansen and Phinney, 1997; Lorimer et al., 1997; Lyne and Lorimer, 1994). It is generally accepted that neutron stars receive a substantial kick at birth, which produce their observed space velocities. However, the physical origin of this kick is unclear. Some of the proposed kick mechanisms require large initial magnetic fields in the magnetar range (10^{14}–10^{16} G) and asymmetric emission of neutrinos (Kusenko and Segrè, 1996; 1997; Lai and Qian, 1998; Maruyama et al., 2011). Other mechanisms require a rapid initial rotation to produce substantial kicks (Khokhlov et al., 1999; Sawai et al., 2008; Spruit and Phinney, 1998). Hydrodynamical models have also been suggested, which are based on recoil due to asymmetric supernovae (Burrows et al., 2007; Burrows and Hayes, 1996; Nordhaus et al., 2012; Scheck et al., 2004). Recent models rely on the existence of topological vector currents (Charbonneau and Zhitnitsky, 2010). The current distribution of the observed neutron star velocities seems to be Maxwellian, which points to a common acceleration mechanism (Hansen and Phinney, 1997; Hobbs et al., 2005). The elusive kick mechanism appears to be connected with the possibility that during some stage of their birth process, neutron stars reach magnetic fields typical of magnetars and periods typical of millisecond pulsars (Duncan and Thompson, 1992; Thompson, 1994; Usov, 1992).

In this paper we suggest that the kick velocity of neutron stars may arise from a magnetorotational instability (MRI) produced at the end of their birth process. More specifically, a newly-born neutron star can be subject to a MRI evolving at the Alfvén time of a few ms in which an emission of radiation energy accompanied by a gain of kinetic energy of translation can be produced at the expense of a loss of kinetic energy of differential rotation. If during the evolving of this birth MRI the period is of a few ms and the magnetic field reaches values of 10^{16} G then we show that the gain of kinetic energy of translation can predict the observed large kick velocity of neutron stars. Our suggestion is supported by studies showing that there is an amplification of the magnetic field and a transference of angular momentum during the evolving of MRI in newly-born neutron stars (Akiyama et al., 2003; Masada et al., 2012; Thompson et al., 2005). Simulations have shown that birth MRI can generate magnetic fields of the order of 10^{16}–10^{17} G in several ms (Siegel et al., 2013; Thompson et al., 2005).

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Newly-born neutron stars are assumed to be highly convective and differentially rotating hot fluids (Stergioulas, 2003; Yamada and Sawai, 2004), which can be subject to MRI evolving at Alfvén timescales (Duez et al., 2006). During this birth MRI the star can convert its kinetic energy of differential rotation into magnetic energy (Akiyama et al., 2003; Spruit, 2008). It is then plausible to assume that the star can lose kinetic energy of differential rotation via radiation (a fraction of the total magnetic energy). Conceivable, in this stage the star can also gain kinetic energy of translation and this is the basic assumption of the model proposed here. We assume that at the end of a birth MRI, neutron stars experience the energy conversion

$$P_{\rm rad} + \frac{d}{dt}\left(\frac{Mv^2}{2}\right) = -\frac{d}{dt}\left(\frac{\alpha_{\rm d}I\Omega^2}{2}\right), \qquad (1)$$

where $P_{\rm rad}$ is the instantaneous radiated power; $Mv^2/2$ is the kinetic energy; $\alpha_{\rm d}I\Omega^2/2$ is the kinetic energy of differential rotation; v is the space velocity; $\alpha_{\rm d}$ is a dimensionless constant parameter accounting for the differential rotation (Spruit, 2008); $I = 2MR^2/5$ is the moment of inertia with M and R being the mass and radius, respectively; and $\Omega = 2\pi/P$ is the angular velocity with P being the period. In Eq. (1) we have assumed that there are not significant changes of the parameter $\alpha_{\rm d}$ during the short period occurring the birth MRI. But in general $\alpha_{\rm d}$ varies with time.

According to Eq. (1) an emission of radiation energy and an increase of kinetic energy of translation occur at the expense of a loss of kinetic energy of differential rotation. We note that an equation similar to Eq. (1) [without $\alpha_{\rm d}$ and with $P_{\rm rad}$ associated with the asymmetric radiation from an off-centered magnetic dipole] is the basis of the "rocket model" proposed by Harrison and Tademaru (1975). Another equation also similar to Eq. (1) [without $\alpha_{\rm d}$ and with $P_{\rm rad}$ estimated by an exponential field decay law] has been considered to study both the birth accelerations of neutron stars (Heras, 2013) and the birth-ultra-fast-magnetic-field decay of neutron stars (Heras, 2012). It is pertinent to say that the idea that newly-born neutron stars would lose their rotational energy catastrophically on a timescale of seconds or less was suggested by Usov (1992).

The instantaneous radiated power $P_{\rm rad}$ in Eq. (1) must express the idea that a MRI is responsible for a rapid exponential growth of the magnetic field (Akiyama et al., 2003; Siegel et al., 2013; Yamada and Sawai, 2004). In this sense, Spruit (2008) has pointed out that some form of MRI occurring during a differential rotation in the final stages of the core collapse phase may produce an exponential growth of the magnetic field and that once formed, the magnetic field is in risk of decaying again by magnetic instabilities. Without considering changes of kinetic energy, it is well-known that abrupt changes of kinetic energy of rotation produce abrupt changes of energy of radiation. The characteristic time of rotation changes is similar to the characteristic time of radiation changes. Therefore if the birth MRI occurs at Alfvén times of ms then the radiative energy must be emitted in these times and therefore the increase and decrease of magnetic fields producing such a radiative energy must occur at these times. Accordingly, we assume here the existence of a birth MRI responsible for a rapid exponential growth of the magnetic field, followed by an equally rapid exponential decay of this field. More explicitly: at the start of the assumed MRI, the magnetic field has the value $B_{\rm A}$ and then it exponentially grows to reach its maximum value $B_{\rm M}$, followed by a rapid exponential decrease reaching the final value $B_{\rm A}$. The exponential growth and decay rates of the magnetic field are assumed to occur with the same characteristic time. In a more general treatment, we can assume that the characteristic times of the field increasing is different from that of the field decaying. However, both times must be of the order of Alfvén times of ms. A similar field behaviour but for an electric field has been discussed in electromagnetism for the decay of the electric dipole moment (Schantz, 1995). Expectably, after this abrupt birth field decay, there will be a subsequent field decay caused
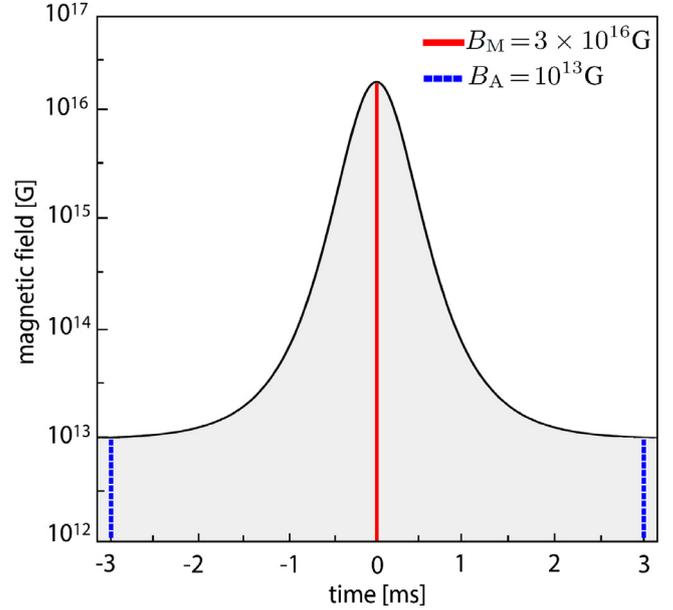

Fig. 1. Qualitative behaviour of the magnetic field of a newly-born neutron star during the assumed MRI. The times $-\tau_{\rm A}/2 = -3$ ms and $\tau_{\rm A}/2 = 3$ ms correspond to the beginning and end of this birth MRI. The value of the magnetic field at the beginning and end of the MRI is $B_{\rm A} = 10^{13}$ G. The maximum value of the magnetic field $B_{\rm M} = 3 \times 10^{16}$ G occurs when $B(0) = B_{\rm M}$.

by Ohmic diffusion and/or other resistive processes, which occur on time scales much larger than those of the birth MRI.

An exponential growth occurs when the growth rate of the function is proportional to the current value of this function: $\dot{f}(t) \propto f(t)$, where the overdot means time differentiation. Analogously, an exponential decay occurs when $\dot{f}(t) \propto -f(t)$. For the case of a magnetic moment $\mu(t)$ the positive and negative growth rates can be described by the relation $\dot{\mu}(t) \propto -\mu(t)\tanh(t/\tau_{\rm a})$, where the hyperbolic tangent function has been introduced to describe the positive and negative growth rates. The time $\tau_{\rm a}$ is the associated characteristic time. Accordingly, the behaviour of the magnetic moment of a newly-born neutron star during a birth MRI is assumed to be described by the equation $\dot{\mu}(t) = -(1/\tau_{\rm a})\mu(t)\tanh(t/\tau_{\rm a})$, whose solution reads

$$\mu(t) = \mu_{\rm M}\,{\rm sech}\,(t/\tau_{\rm a}), \qquad (2)$$

where $\mu_{\rm M}$ is the maximum value of the magnetic dipole moment satisfying $\mu(0) = \mu_{\rm M}$. For a neutron star of radius R, the magnetic moment μ is related to the magnetic field B by means of $\mu(t) = B(t)R^3$ (Jackson, 1998), which can be used together with Eq. (2) to obtain the expected magnetic field law

$$B(t) = B_{\rm M}\,{\rm sech}\,(t/\tau_{\rm a}), \qquad (3)$$

where $B_{\rm M} = B(0)$ is the maximum value of the magnetic field. The behaviour of the magnetic field in the assumed birth MRI is qualitatively shown in Fig. 1.

Eq. (2) implies the non-linear equation $\ddot{\mu}^2 - 4(\dot{\mu}^4/\mu^2 - \dot{\mu}^2/\tau_{\rm a}^2) - \mu^2/\tau_{\rm a}^4 = 0$, which combines with $\mu(t) = B_{\rm M}R^3\,{\rm sech}\,(t/\tau_{\rm a})$ to yield

$$\ddot{\mu}^2(t) = \frac{B_{\rm M}^2 R^6}{\tau_{\rm a}^4}\,{\rm sech}\left(\frac{t}{\tau_{\rm a}}\right)^2\left[2\tanh\left(\frac{t}{\tau_{\rm a}}\right)^2 - 1\right]^2. \qquad (4)$$

Using the Larmor formula $P_{\rm rad}(t) = 2\ddot{\mu}(t)^2/(3c^3)$ and Eq. (4), we obtain the instantaneous radiated power by a newly-born neutron star during the birth MRI,

$$P_{\rm rad} = \frac{2B_{\rm M}^2 R^6}{3c^3\tau_{\rm a}^4}\,{\rm sech}\left(\frac{t}{\tau_{\rm a}}\right)^2\left[2\tanh\left(\frac{t}{\tau_{\rm a}}\right)^2 - 1\right]^2. \qquad (5)$$

The change of kinetic energy of differential rotation can be written as $d(\alpha_d I \Omega^2/2)/dt = (4\alpha_d \pi^2 M R^2/5)d(1/P^2)/dt$. Using this relation together with Eqs. (1) and (5) we obtain the instantaneous energy conversion occurring in the birth MRI:

$$\frac{2B_M^2 R^6}{3c^3 \tau_a^4} \text{sech}\left(\frac{t}{\tau_a}\right)^2 \left[2\tanh\left(\frac{t}{\tau_a}\right)^2 - 1\right]^2 + \frac{d}{dt}\left[\frac{Mv^2(t)}{2}\right]$$

$$= -\frac{4\alpha_d \pi^2 M R^2}{5}\frac{d}{dt}\left[\frac{1}{P^2(t)}\right]. \tag{6}$$

Let us consider the time τ_A during which the MRI evolves. We will call τ_A the MRI time. We adopt the conditions $B(\pm\tau_A/2) = B_A$, where B_A is the value of the magnetic field at the beginning and end of the birth MRI. These conditions and the law in Eq. (2) imply the result $B_A = B_M \text{sech}[\tau_A/(2\tau_a)]$ or equivalently

$$\tau_A = 2\tau_a \text{asech}(B_A/B_M), \tag{7}$$

where asech denotes the inverse hyperbolic secant function.

Integration of Eq. (6) over the time interval in which the assumed birth MRI evolves: $-\tau_A/2 \leq t \leq \tau_A/2$, or equivalently, $-\tau_a \text{asech}(B_A/B_M) \leq t \leq \tau_a \text{asech}(B_A/B_M)$, yields

$$\underbrace{\frac{4}{45}\frac{R^6(B_M^2 - B_A^2)^{1/2}(7B_M^4 + 12B_A^4 - 4B_M^2 B_A^2)}{B_M^3 c^3 \tau_a^3}}_{\Delta E_{rad}} + \underbrace{\frac{Mv^2}{2}}_{\Delta E_{kin}}$$

$$= \underbrace{\frac{4\alpha_d \pi^2 M R^2}{5}\left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right)}_{\alpha_d \Delta E_{rot}}, \tag{8}$$

where $P_a = P(-\tau_A/2)$ and $P_b = P(+\tau_A/2)$ are the values of the period at the beginning and the end of the birth MRI, respectively. The quantity $v = v(\tau_A/2)$ denotes the space velocity at the end of this instability, which is assumed to be the current value of the observed kick velocity. The initial condition $v(-\tau_A/2) = 0$ has been adopted, which is consistent with the idea that newly born stars acquire their space velocity during the birth MRI.

If $B_M \gg B_A$ then the radiation term in Eq. (8) reduces to $\Delta E_{rad} \approx (28/45)R^6 B_M^2/(c^3 \tau_a^3)$ and thus Eq. (8) becomes

$$\frac{28}{45}\frac{R^6 B_M^2}{c^3 \tau_a^3} + \frac{Mv^2}{2} = \frac{4\alpha_d \pi^2 M R^2}{5}\left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right). \tag{9}$$

We expect that the time τ_A is of a few ms for magnetic fields of the order of 10^{16} G. This expectation restricts the values of the time τ_a and those of the relation B_A/B_M. In particular, if τ_a is taken to be $\tau_a \approx 10R/c$ then Eq. (7) implies a MRI time τ_A in the interval $3 \text{ ms} \lesssim \tau_A \lesssim 6 \text{ ms}$ when $B_M = 3 \times 10^{16}$ and B_A lies in the interval $10^{13} \text{ G} \leq B_A \leq 10^{15}$ G. The choice $\tau_a \approx 10R/c$ is then consistent with the model proposed here. Insertion of $\tau_a \approx 10R/c$ into Eq. (9) yields

$$\frac{7R^3 B_M^2}{11250} + \frac{Mv^2}{2} = \frac{4\alpha_d \pi^2 M R^2}{5}\left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right). \tag{10}$$

Let us emphasize that Eq. (10) is valid when the magnetic field B_M is given by Eq. (3) and the conditions $B_M \gg B_A$ and $\tau_a \approx 10R/c$ are fulfilled. Eq. (10) can be used to constrain the values of the periods P_a and P_b. In fact, Eq. (10) with $\alpha_d = .1$, $M = 1.4M_\odot$ and $R = 10$ km imply

$$P_b = \frac{1}{\sqrt{P_a^{-2} - (\kappa_1 B_M^2 + \kappa_2 v^2)}}, \tag{11}$$

where $\kappa_1 \approx .281 \times 10^{-30}$ cm/gr and $\kappa_2 \approx .633 \times 10^{-12}$ cm^{-2}. Since P_b is real it follows that

$$P_a < \frac{1}{\sqrt{\kappa_1 B_M^2 + \kappa_2 v^2}}. \tag{12}$$

This condition restricts the values of B_M and v. By assuming $B_M = 3 \times 10^{16}$ G, we will apply Eqs. (11) and (12) to three different velocities: (a) If $v = 100$ km/s then Eq. (12) implies $P_a < .0561$ s. In particular, if $P_a = .015$ s then Eq. (11) gives $P_b \approx .0155$ s; (b) If $v = 500$ km/s then $P_a < .0233$ s. For example, if $P_a = .01$ s then $P_b \approx .0108$ s; and (c) If $v = 1000$ km/s then $P_a < .0123$ s. For instance, if $P_a = .005$ s then $P_b \approx .0055$ s. If the initial period satisfies $P_a \leq .015$ s then the changes $\Delta P = P_b - P_a$ (for the previous examples) are of the order of 10^{-4} s for the interval of space velocities 100 km/s $\leq v \leq$ 1000 km/s.

We can use Eq. (10) to obtain a formula for the kick velocity

$$v_{kick} = \sqrt{\frac{8\alpha_d \pi^2 R^2}{5}\left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right) - \frac{7R^3 B_M^2}{5625M}}, \tag{13}$$

where we have written $v \equiv v_{kick}$ to emphasize that the kick velocity of neutron stars is originated by the birth MRI quantities: α_d, B_M, P_a and P_b. This result explains the absence of the suspected correlation between the current magnetic field and space velocity, the so-called "v–B correlation" (Cordes and Chernoff, 1998; Lorimer et al., 1997; 1995). Let us rewrite Eq. (13) as

$$v_{kick} = \sqrt{v_{[drot]}^2 - v_{[rad]}^2}, \tag{14}$$

where $v_{[drot]}$ is the component of the kick velocity originated by the change of kinetic energy of differential rotation

$$v_{[drot]} = \pi R\sqrt{\frac{8\alpha_d}{5}\left(\frac{1}{P_a^2} - \frac{1}{P_b^2}\right)}, \tag{15}$$

and $v_{[rad]}$ is the component of the kick velocity originated by the change of radiation energy

$$v_{[rad]} = \frac{B_M}{75}\sqrt{\frac{7R^3}{M}}. \tag{16}$$

We can infer the kick force $F_{kick}(t)$ producing $v_{kick}(t)$. Time-integration of Eq. (6) from $-\tau_A/2$ to the time t leads to

$$v_{kick}(t) = \sqrt{v_{[drot]}^2(t) - v_{[rad]}^2(t)}, \tag{17}$$

where

$$v_{[drot]}(t) = \sqrt{\kappa\left[\frac{1}{P_a^2} - \frac{1}{P(t)^2}\right]}, \tag{18}$$

in which $\kappa = (8\alpha_d \pi^2 R^2)/5$, $P_a = P(-\tau_A/2)$, and

$$v_{[rad]}(t) = \sqrt{\frac{2}{M}\int_{-\tau_A/2}^{t} P_{rad}(\sigma)d\sigma}. \tag{19}$$

If Eq. (18) is evaluated in the time $t = \tau_A/2$ and we write $P_b = P(\tau_A/2)$ then we obtain Eq. (15). Analogously, if Eq. (19) with P_{rad} given by Eq. (5) is evaluated in $t = \tau_A/2$ and the conditions $B_M \gg B_A$ and $\tau_a \approx 10R/c$ are fulfilled then we obtain Eq. (16). Using Eqs. (18) and (19) we can derive the corresponding accelerations. They can be expressed as

$$a_{[drot]}(t) = \frac{\kappa \dot{P}(t)}{P^3(t)v_{[drot]}(t)}, \quad a_{[rad]}(t) = \frac{P_{rad}(t)}{v_{[rad]}(t)}. \tag{20}$$

Time differentiation of Eq. (17) and the use of Eq. (20) gives the kick (total) acceleration: $a_{[kick]} = v_{[drot]}a_{[drot]}/v_{[kick]} - v_{[rad]}a_{[rad]}/v_{[kick]}$, which can alternatively be expressed as

$$a_{[kick]}(t) = \frac{\kappa \dot{P}(t)}{P(t)^3 v_{[kick]}(t)} - \frac{P_{rad}(t)}{Mv_{[kick]}(t)}. \tag{21}$$

We can evaluate $a_{[kick]}$ at $t = 0$ to get an idea of the order of its magnitude. According to Eq. (5), the power radiated at $t = 0$ is $P_{rad}(0) \approx .1874 \times 10^{52}$ ergs for $B_M = 3 \times 10^{16}$ G, $R = 10$ km and $\tau_a = 3.33 \times$

10^{-4} s. From Eq. (17) it follows that $v_{[kick]}(0) \approx 128$ km/s if we assume $P_a = .019$ s, $P(0) = .01986$ s and $M = 1.4 M_\odot$. We write $\kappa \approx .158 \times 10^{13}$ cm^2 (for $\alpha_d = 0.1$ and $R = 10$ km) and $\dot{P}(0) \approx 6.62$ (calculated by $\dot{P} \sim 2P/\tau_A$). Using all of these values in Eq. (21) we get: $a_{[kick]}(0) \approx .53 \times 10^8$ g (Heras, 2013).

The forces associated with accelerations in Eqs. (20) read

$$F_{[drot]}(t) = \frac{M\kappa \dot{P}(t)}{P^3(t) v_{[drot]}(t)}, \quad F_{[rad]}(t) = \frac{M P_{rad}(t)}{v_{[rad]}(t)}. \tag{22}$$

By making use of $F_{[kick]} = (v_{[drot]} F_{[drot]} - v_{[rad]} F_{[rad]})/v_{[kick]}$ and Eqs. (22) we can obtain the kick (total) force

$$F_{[kick]}(t) = \frac{\kappa M \dot{P}(t)}{P(t)^3 v_{[kick]}(t)} - \frac{P_{rad}(t)}{v_{[kick]}(t)}. \tag{23}$$

It should be noted that Eq. (23) can naturally be interpreted in the context of the radiation reaction theory. If $Ma = F_{[kick]}$; $\mathcal{F}_{ext} = \kappa M \dot{P}/(P^3 v_{[kick]})$; and $\mathcal{F}_{rad} = -P_{rad}/v_{[kick]}$ then we obtain the well-known equation for the radiation reaction (Jackson, 1998): $Ma = \mathcal{F}_{ext} + \mathcal{F}_{rad}$.

The velocity $v_{[rad]}$ in Eq. (16) can be related to the Alfvén velocity $v_{[Alf]} = B/\sqrt{4\pi\rho}$. For a sphere of mass M and radius R the Alfvén velocity reads $v_{[Alf]} = B/\sqrt{3M/R^3}$, which combines with Eq. (16) (making $B = B_M$) to yield $v_{[rad]} = \sqrt{21} v_{[Alf]}/75$. This implies $v_{[Alf]} \approx 16.37 v_{[rad]}$. If $v_{[rad]} \approx 200$ km/s for $B_M = 3 \times 10^{16}$ G, $M = 1.4 M_\odot$ and $R = 10$ km then $v_{[Alf]} \approx 3.27 \times 10^8$ cm/s, and this can be combined with $v_{[Alf]} = 2R/T_A$ (considering $R = 10$ km) to get the expected Alfvén time $T_A \approx 6$ ms, which emphasizes the consistency of the model proposed here.

Let us apply Eqs. (14)–(16) to two representative neutron stars: The Crab pulsar B0531+21 and the magnetar J1809-1943. The canonical values $M = 1.4 M_\odot$ and $R = 10$ km will be assumed. Unless otherwise specified, all data is taken from the ATNF Pulsar Catalogue (Manchester et al., 2005). The space velocity v is obtained from the transverse velocity $v_\perp$ by means of $v \approx \sqrt{3/2}\, v_\perp$ (Hobbs et al., 2005; Lyne and Lorimer, 1994).

Consider first the Crab pulsar with its current transverse velocity $v_\perp = 141$ km/s, which implies the space velocity $v \approx 172$ km/s. It has been suggested that this pulsar was born with a period of 19 ms (Lyne et al., 1993). If during a birth MRI in the Crab pulsar there was a very small increase in the period, for example, from $P_a = .019$ s to $P_b = .02072$ s and an abrupt change of magnetic field starting with the value 10^{13} G, reaching the maximum value 3×10^{16} G and ending with the value 10^{13} G, then Eq. (14) with $\alpha_d = 0.1$ yields $v_{kick} \approx 172$ km/s and Eqs. (15) and (16) respectively give $v_{[drot]} \approx 264$ km/s and $v_{[rad]} \approx 200$ km/s. Notice that the difference between the periods P_a and P_b is small: $\Delta P \approx .001$ s. The current period of the Crab pulsar is $P = .033$ s. Crab's period has increased about 13 ms since the occurrence of the assumed birth MRI.

Consider now the magnetar J1809-1943, whose current period and magnetic field are $P = 5.54$ s and $B = 2.1 \times 10^{14}$ G. This magnetar has $v_\perp = 229$ km/s, which implies $v \approx 278$ km/s. If this magnetar experienced a birth MRI in which there was a very small increase of its period, for example, from the value $P_a = .010$ s to $P_b = .01039$ s and an abrupt change of magnetic field starting with the value 10^{15} G, reaching the maximum value 3×10^{16} G and ending with the value 10^{15} G, then Eq. (14) with $\alpha_d = 0.1$ yields $v_{kick} \approx 278$ km/s and Eqs. (15) and (16) respectively give $v_{[drot]} \approx 342$ km/s and $v_{[rad]} \approx 200$ km/s.

It is interesting to note that Eq. (11) allows the final period P_b to have values considerably larger than those previously assumed. This possibility could explain why some neutron stars exhibit current periods in the scale of seconds (e.g., magnetars). According to

Eq. (11) the period P_b may be of the same order of the current period P. Consider, for example, the magnetar SGR 1806-20 whose current period and magnetic field are $P = 7.54$ s and $B = 2 \times 10^{15}$ G. This magnetar has $v_\perp \approx 350$ km/s (Olausen and Kaspi, 2014) which implies $v \approx 428$ km/s. If we assume the birth MRI values $B_A = 4 \times 10^{15}$ G and $P_a = .0265996$ s then Eq. (11) implies $P_b = 6.01$ s. This result suggests that shortly after the evolving of the assumed birth MRI, this magnetar acquired a period of the order of seconds. However, this conclusion faces the problem of how to physically explain the birth MRI change $\Delta P = P_b - P_a$ of the order of seconds.

In summary, any convincing explanation for the observed large space velocity of neutron stars should be traced to physical processes occurring during their birth. Accordingly, the space velocity should be related with birth values of physical properties of these stars and not with current values of them. Here we have suggested that a MRI produced at the end of the birth process of neutron stars can be responsible for their large space velocities. We have shown how a rapid birth MRI conversion of kinetic energy of differential rotation into radiation energy and kinetic energy of translation occurring at the Alfvén time of a few ms yields an observed interval for neutron star velocities, ranging from several hundreds to a few thousands km/s.

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